膜構造研究論文集 第30号, 9-14, 2016年
Research Paper on Membrane Structures, No.30, 9-14, 2016

Nonlinear finite element procedure for membrane structures using ETFE films considering friction contact condition

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Synopsis

In frame-supported membrane structures type, one of techniques to introduce the pre-stress on the membranes is extending the boundary. In this case, the friction contact between the membrane and the frame-supported, such as steel arch, was observed. Therefore, the analytical method of membrane structures considering the friction contact is essential.

This paper describes a nonlinear finite element procedure, which includes the non-linear geometry of membrane elements and friction contact condition. In this analysis, the friction contact between membrane elements and frame-supported elements was replaced by Gap elements. The proposed method will be applied to investigate the sliding between ETFE film and steel arch of the parallel steel arch model under forced displacement condition. The accuracy and applicability of proposed procedure was confirmed by an experiment.

1. Introduction

Recently, ETFE film is well used in membrane structures because its advantages, such as transparent, light weight, easy to fabricate a large-sized panel by heat-sealing and excelling in the durability over ultraviolet rays. In term of design of ETFE membranes structure, since the ETFE film possesses no flexural stiffness, its shape, the load on structure and the internal stresses interact in a nonlinear manner to satisfy the equilibrium equations. Therefore, various kinds of analysis procedure have been proposed to solve those geometrical nonlinear problems. One of them was the finite element technique. The procedure of this technique was described clearly in many previous reports [1][2].

In term of suspension and frame-supported ETFE membrane structures, one of techniques to introduce the pre-stress on the membranes is extending the boundary. In this case, the friction contact between the films and the frame-supported was observed. Therefore, the analytical method of membrane structures considering the friction contact is essential. Computationally, the analysis of frictional contact problems is extremely difficult, even for static cases involving the simplest constitute relation [3]. The reasons could be explained as the boundary conditions are unknown as a priori and the contacting surfaces change in size and shape as load is applied. There had two main approaches to solve the contact problem in FEM analysis, which are the Lagrange multiplier method and Penalty method. The later method were attracted researchers to develop the Gap/Friction element. Contact element stiffness matrices are symmetric for frictionless contact problems. Unfortunately, problems involving sliding friction result in an unsymmetrical stiffness matrix which makes the problems in convergence criteria. In 1995 Ju et al suggested the symmetrical stiffness matrix for 3D frictional contact element which can be implemented into an existing nonlinear finite element program to solve the nonlinear friction contact [4][5].

In this paper, the FEM procedure which includes the non-linear geometry of membrane elements and nonlinear friction contact problem using Ju's method, is proposed. The proposed method will be applied to investigate the sliding between ETFE film and steel arch of the parallel steel arch model under forced displacement condition. The accuracy and applicability of proposed procedure was confirmed by an experiment.

2. Finite element formulation

2.1 Membrane elements

The triangular membrane elements with three nodes and nine degrees of freedom per element were used in this paper. The equilibrium equation for a single element in local coordinate system may be obtained via the principle of virtual work, as follows:

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$$\int_{V^e} \delta\boldsymbol{\epsilon}^{\mathrm{T}}\boldsymbol{\sigma}\mathrm{dV} - \delta\mathbf{u}^{\mathrm{T}}\mathbf{p} = \mathbf{0} \tag{2.1}$$

where, $\boldsymbol{\sigma}$: stress, $\mathbf{p}$: the external node force vector, $\boldsymbol{\epsilon}$: strain, $\mathbf{u}$: the node displacement vector, $\mathbf{0}$: zeroes vector and V^e : the element volume. The relation between the strain $\boldsymbol{\epsilon}$ and node displacement vector $\mathbf{u}$ can be described as follows:

$$\boldsymbol{\epsilon} = \mathbf{B}_0\mathbf{u} + 0.5\mathbf{A}\boldsymbol{\theta}; \quad \delta\boldsymbol{\epsilon} = (\mathbf{B}_0 + \mathbf{AG})\delta\mathbf{u} \tag{2.2}$$

here, the matrices $\mathbf{B}_0, \mathbf{A}, \boldsymbol{\theta}, \mathbf{G}$ can be referenced in [2]. In this paper, the large displacements and small strains are considered, so the constitute relations for plane stress analysis may be used as:

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\epsilon} + \boldsymbol{\sigma}_0 \tag{2.3}$$

where, $\boldsymbol{\sigma}_0$ denotes the initial stress vector and $\mathbf{D}$ is the elastic matrix defined as:

$$\mathbf{D} = \frac{E}{1-\vartheta^2}\begin{bmatrix} 1 & \vartheta & 0 \\ \vartheta & 1 & 0 \\ 0 & 0 & \frac{1-\vartheta}{2} \end{bmatrix}$$

here, E and ϑ are the Young's modulus and Poisson's ratio.

Substituting (2.2) and (2.3) into (2.1) and eliminating $\delta\mathbf{u}^{\mathrm{T}}$, we have:

$$\int_{V^e} (\mathbf{B}_0 + \mathbf{AG})^{\mathrm{T}}[\mathbf{D}(\mathbf{B}_0\mathbf{u} + 0.5\mathbf{A}\boldsymbol{\theta}) + \boldsymbol{\sigma}_0]\mathrm{dV} - \mathbf{p} = \mathbf{0} \tag{2.4}$$

The above equations must be transformed to global coordinates and finally assembled to obtain the global equilibrium equations. Since the global equations will be solved iteratively by Newton-Raphson method, the linearize process of the governing equation, at the element level, can be described after the *i*th iteration as follow:

$$\mathbf{k}_m^i\Delta\mathbf{u}^i = \mathbf{p} - \mathbf{r}^i \tag{2.5}$$

here, the node force vector after *i*th iteration $\mathbf{r}^i$ can be determined by (2.6):

$$\mathbf{r}^i = \int_{V^e} (\mathbf{B}_0 + \mathbf{A}^i\mathbf{G})^{\mathrm{T}}[\mathbf{D}(\mathbf{B}_0\mathbf{u}^i + 0.5\mathbf{A}^i\boldsymbol{\theta}^i) + \boldsymbol{\sigma}_0]\mathrm{dV} \tag{2.6}$$

and the element tangent stiffness matrix, $\partial\mathbf{r}^i/\partial\mathbf{u}$, consists of two parts as (2.7):

$$\mathbf{k}_m^i = \int_{V^e} (\mathbf{B}_0 + \mathbf{A}^i\mathbf{G})^{\mathrm{T}}\mathbf{D}(\mathbf{B}_0 + \mathbf{A}^i\mathbf{G})\mathrm{dV} + \int_{V^e} \mathbf{G}^{\mathrm{T}}\mathbf{M}^i\mathbf{G}\mathrm{dV} \tag{2.7}$$

here, the matrix $\mathbf{M}$ can be also referenced in [2].

2.2 Gap elements

A Gap element is considered to connect nodes A and B. The local coordinate xyz of this element is shown in Fig. 1. The normal direction of Gap element is assumed in z direction, while the sliding direction, when Gap element is in sliding status, is assumed in x direction. The y direction is determined from x and z directions

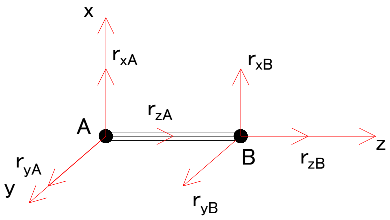

Fig.1 The local coordinate of Gap elements

The relation between the internal node force vector and incremental displacement vector for Gap element via the penalty method can be determined as follows:

$$\mathbf{r} = \mathbf{k_g}\Delta\mathbf{u} \tag{2.8}$$

here, $\Delta\mathbf{u} = [\Delta u_{\boldsymbol{A}} \quad \Delta v_{\boldsymbol{A}} \quad \Delta w_A \quad \Delta u_{\boldsymbol{B}} \quad \Delta v_{\boldsymbol{B}} \quad \Delta w_{\boldsymbol{B}}]^{\mathbf{T}}$ is incremental node displacement vector, in which Δu, Δv and Δw are incremental displacements in x, y and z directions, respectively; $\mathbf{r} = [r_{xA} \quad r_{yA} \quad r_{zA} \quad r_{xB} \quad r_{yB} \quad r_{zB}]^{\mathbf{T}}$ is the internal node force vector of Gap element; and $\mathbf{k_g}$ is Gap element stiffness matrix and determined as equation (2.9) [4]:

$$\mathbf{k_g} = \begin{bmatrix} \mathbf{k} & -\mathbf{k} \\ -\mathbf{k} & \mathbf{k} \end{bmatrix} \tag{2.9}$$

where, $\mathbf{k} = \begin{bmatrix} k_s & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_n \end{bmatrix}$ is used when Gap element is in stick status and $\mathbf{k} = \begin{bmatrix} \mu^2 k_n & 0 & \mu k_n \\ 0 & k_s & 0 \\ \mu k_n & 0 & k_n \end{bmatrix}$ is used when Gap element is in sliding status; k_n is an input large penalty constant for normal direction of Gap element; k_s is the penalty constant for sliding directions and μ is the static friction coefficient.

In sliding status, because the assumption of sliding direction is x direction, the internal friction force $f_s = r_{xA} = -r_{xB}$ and the normal force $f_n = r_{zA} = -r_{zB}$ obey the Mohr-Coulomb friction theory ($f_s = \mu f_n$) according to the equations (2.8)&(2.9); however the increment of normal force f_n contains the term, $\mu k_n(\Delta u_{\boldsymbol{A}} - \Delta u_{\boldsymbol{B}})$. If this term is omitted from the calculation of the total normal force, the contact analysis from equations (2.8)&(2.9) will produce a similar result at the convergence state ($\Delta u_{\boldsymbol{A}} - \Delta u_{\boldsymbol{B}} \cong 0$).

2.3. Membrane structures include Gap elements

The principle of virtual work for the membrane structure include Gap elements can be expressed as:

$$\int_{V} (\delta\boldsymbol{\epsilon}_m)^{\mathrm{T}}\boldsymbol{\sigma}_m \mathrm{dV} + \delta\mathbf{U}^{\mathrm{T}}\mathbf{R}_g - \delta\mathbf{U}^{\mathrm{T}}\mathbf{P} = \mathbf{0} \qquad (2.10)$$

where, $\boldsymbol{\epsilon}_m$ and $\boldsymbol{\sigma}_m$ are the strain and stress of membrane elements; $\mathbf{R_g}$ is internal force of Gap elements; $\mathbf{U}$ is the whole structure node displacement vector and $\mathbf{P}$ is the external node vector.

After eliminating $\delta\mathbf{U}^{\mathrm{T}}$ and solving the global equation (2.10) by Newton-Raphson method, the governing equation after the *i*th iteration can be described as:

$$(\mathbf{K}_{\boldsymbol{m}}^{\boldsymbol{i}} + \mathbf{K}_{\boldsymbol{g}}^{\boldsymbol{i}})\Delta\mathbf{U}^{\boldsymbol{i}} = \mathbf{P} - \mathbf{R}_{\boldsymbol{m}}^{\boldsymbol{i}} - \mathbf{R}_{\boldsymbol{g}}^{\boldsymbol{i}} \qquad (2.11)$$

here, $\mathbf{K}_{\boldsymbol{m}}^{\boldsymbol{i}}$ and $\mathbf{R}_{\boldsymbol{m}}^{\boldsymbol{i}}$ are the tangent stiffness matrix and internal force vector of membrane elements, which calculated by assembly the elements stiffness in equation (2.7) and the element internal force in equation (2.6) respectively; $\mathbf{K}_{\boldsymbol{g}}^{\boldsymbol{i}}$ and $\mathbf{R}_{\boldsymbol{g}}^{\boldsymbol{i}}$are the tangent stiffness matrix and internal force vector of Gap elements, which calculated by assembly the elements stiffness in equation (2.9) and the element internal force in equation (2.8) respectively depend on the status of elements.

The displacements after *(i+1)*th iteration can be determined as:

$$\mathbf{U}^{\boldsymbol{i}+\mathbf{1}} = \mathbf{U}^{\boldsymbol{i}} + \Delta\mathbf{U}^{\boldsymbol{i}} \qquad (2.12)$$

This updating of the node point displacements in the iteration is continue until the incremental displacements, $\Delta\mathbf{U}^{\boldsymbol{i}}$, are small or nearly zeroes. This convergence criteria will satisfy the condition of using the symmetric stiffness matrix of Gap elements in case of sliding status.

3. Analysis procedures

The analysis procedures for membrane structures include Gap elements are listed below:

(a) Input data

(b) Assume all Gap elements are in stick condition for the first loop of Newton-Raphson method

(c) Determinate the structure stiffness matrix at step i: $\mathbf{K}_{\boldsymbol{m}}^{\boldsymbol{i}}$+$\mathbf{K}_{\boldsymbol{g}}^{\boldsymbol{i}}$

(d) Determinate the internal force vector at step i: $\mathbf{R}_{\boldsymbol{m}}^{\boldsymbol{i}} + \mathbf{R}_{\boldsymbol{g}}^{\boldsymbol{i}}$

(e) Determinate the incremental displacement vector by Eq. (2.11)

(f) Check the Gap elements status: the Gap elements will be assumed in stick status until equation (2.13) is reached. And it is assumed that if Gap elements change into sliding status, this status will be remained till the end of Newton-Raphson loop:

$$\sqrt{r_x^2 + r_y^2} - \mu|r_z| = 0 \qquad (2.13)$$

here, $r_x = r_{xA} = -r_{xB}$ and $r_y = r_{yA} = -r_{yB}$ are the friction forces in x and y directions and $r_z = r_{zA} = -r_{zB}$ is the normal force in z direction of Gap elements in stick condition, which are determined by Eq. (2.8)

(g) Check the convergence for the incremental displacement $\Delta\mathbf{U}^{\boldsymbol{i}}$

(h) If step (g) converges, then output the results

(i) If step (g) does not converge, then return to step (c) for the next iteration.

4. Analysis examples

This numerical example is based on the experiments of new method of form-finding for ETFE film structure.

4.1. Experiments overview

For the tension type of ETFE membrane structure, the films are pulled outward from the exterior to introduce membrane tension. In those experiments, the stretching boundary of ETFE film with two parallel steel arch model were took place. The dimension of specimens are described as Fig. 2. There are three type of arch height H (200mm, 300mm and 400mm), which determined depend on the span of outside frame.

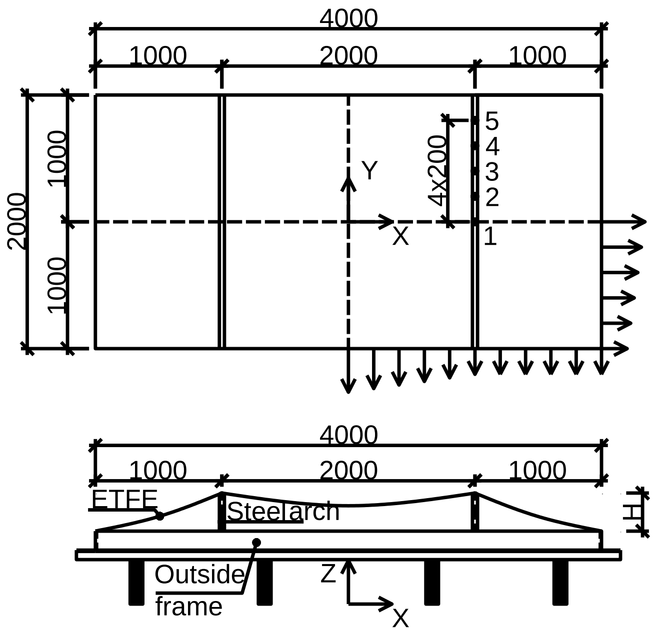

Fig.2 Specimens overview (all dimensions in millimeter)

First, the outside frames and steel arches were fixed first. After that, the flatting ETFE film was put on the steel arches and outside frame. After the ETFE film was fixed with outside frame, the forced displacement was input in Z direction step by step until getting the target values. During the experiments, the movement of offset points on ETFE films in X and Y direction, which shown in ¼ model of specimens in Fig. 2, were observed and were used for the forced displacement condition in analysis. Those value can be referred in [6].

In addition, during the experiments, the sliding between the ETFE films and steel arches were also observed at special points (from 1 to

5) in Fig. 2. Those experiments results were shown in Table 1 at the final step of experiments

Table 1. Experiments results on sliding observation

Specimens	Sliding distances on arch direction (mm)				
	Observed points				
	1	2	3	4	5
200_Spec	0.0	-1.0	2.5	4.5	6.5
300_Spec	0.0	6.0	11.0	16.5	21.0
400_Spec	1.0	8.0	15.0	21.0	27.0

4.2. Input data of analysis examples

In this analysis examples, the triangular elements will be used to model the ETFE membrane. And the friction contact between ETFE films and steel arch was modeled by Gap elements. The formulations of those elements were described in section 2 and section3. The general view of this analysis will be shown in Fig. 3. However because of the symmetric, ¼ analysis model is used to save the time of calculation.

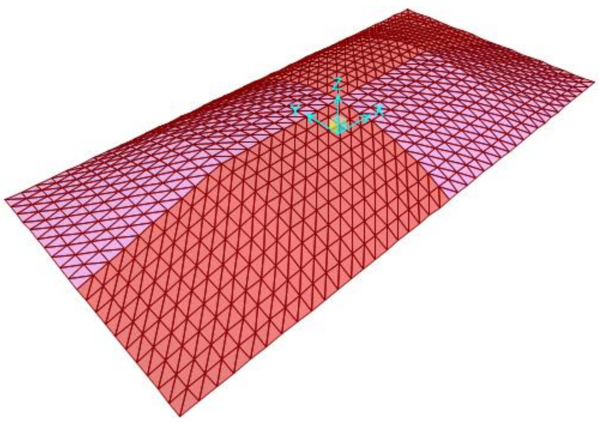

Fig 3. The general view of analysis model

The Fig. 4 shows the dimension of ¼ analysis model, the position of Gap elements and the boundary conditions. In this model, two hundreds triangular ETFE membrane elements were used. The contacts between ETFE membrane and steel arch were replaced by nine Gap elements at arch position. The input forced displacements at fixed nodes (1*→31*) were determined from the experiments [6].

4.3. The assumption of local coordinate of Gap elements

In this analysis, the normal direction (z axis) of Gap element is assumed from the center of the arch to contact points. The sliding direction (x axis) is assumed as the tangent direction of arch in point contacts. The y direction is determined from x and z direction. This local coordinate of Gap elements is assumed unchanged during the calculation. The Fig. 5 illustrates the local coordinate of Gap elements.

4.4 Results and discussions

For penalty method, the large penalty constant which can achieve convergence is strived to find. The very large penalty constant usually yields an oscillated numerical solution; however the small penalty constant typically results in easy convergence, but the numerical results might be less reliable. In this paper, the recommendation of S.H. Lee [7] for those value were used. The normal stiffness penalty value k_n was get as three high order than ETFE films, and the lateral stiffness penalty value k_s was get as 0.1 k_n.

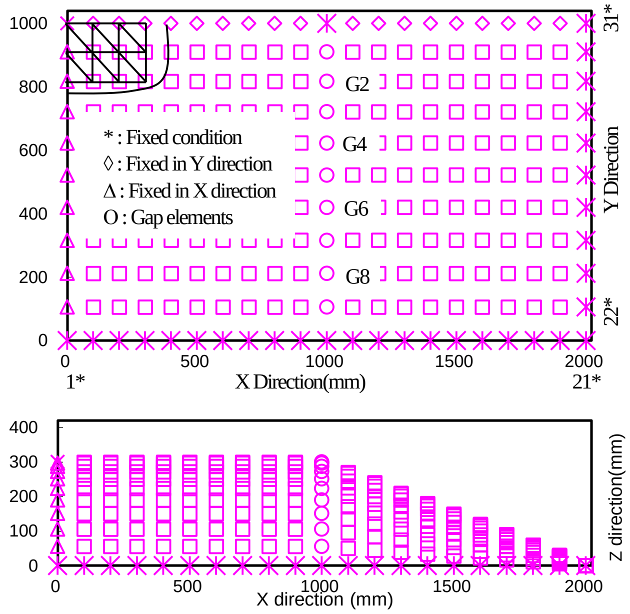

Fig. 4. Initial nodes coordinates for the modeling in case of 300mm Arch model

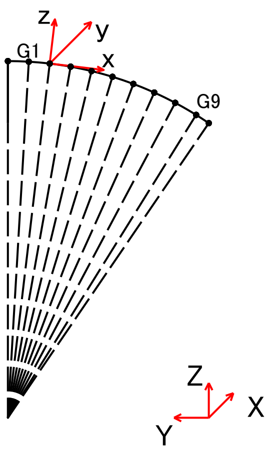

Fig. 5. The assumptions of local coordinate of Gap elements

The equivalent stresses on membrane elements and the deformation shape, when static friction coefficient was chosen as 0.2 in case of 300mm arch model, were shown in Fig. 6. As it can be seen from Fig. 6, the sliding between the ETFE film and steel arch can be only seen in arch direction. The reason can be explained as the assumption of local coordinate of Gap elements in section 4.3. Although this results is different with experiment results, this assumption still be used for the next investigation because the sliding in arch direction is mainly observed in experiments.

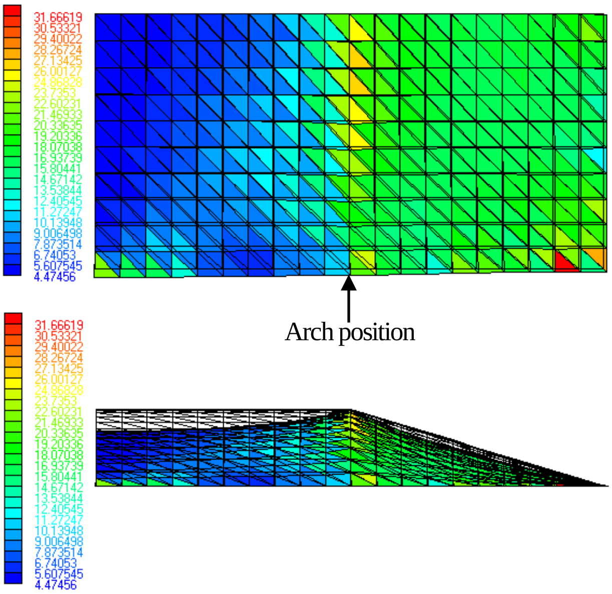

Fig. 6 The equivalent stress on membrane elements (N/m²)

A various values of static friction coefficients were used in order to illustrate the accuracy and applicability of above proposed procedure. In case of 300mm arch model, the figure 7 shows the sliding values between ETFE films and steel arch at 9 Gap elements positions in Fig 2. As we can see in Fig. 7, the larger friction coefficient the smaller sliding values. Those analysis results were completely coincide with the physically phenomenon.

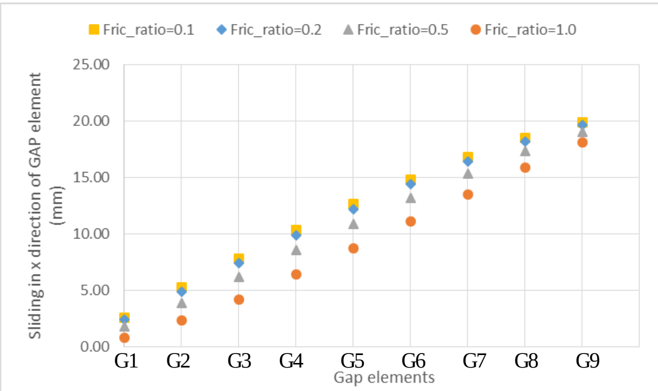

Fig. 7 The sliding results in x direction of Gap elements 300mm arch model

In many technical reports for the properties characteristics of ETFE films, the friction coefficient between ETFE films and steel was described as 0.2 [8]. Therefore, this value was used for the next investigation. The figure 8 shows the sliding values in arch direction in both experiments and analyses. The dash line shows the experiments results while the solid line shows the analysis results. Four observed point (2→5) in experiments in Fig. 2 were corresponding with G2, G4, G6 and G8 Gap elements in analysis in Fig. 4.

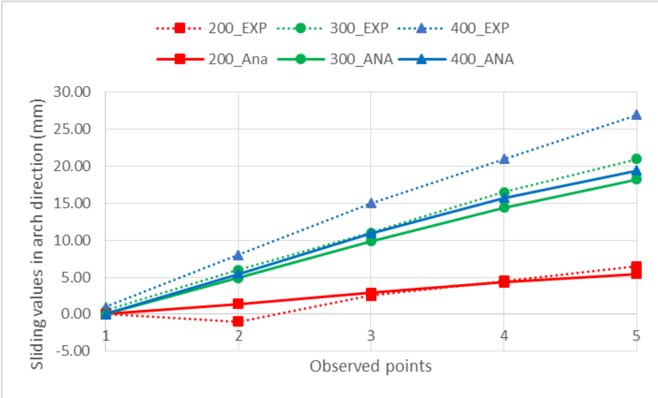

Fig. 8 The sliding results of experiments and analysis

As we can see in Fig. 8, the analysis results and experiment results were almost the same in case of *200mm* and *300mm* arch models. In case of *400mm* arch model, the large difference between analysis and experiment was observed. The reason can be explained as the sliding in analysis was observed in x local direction of Gap elements as the tangent direction of arch, while in the experiment the sliding value was measured on the curve of arch.

5. Conclusions

In this research, the friction contact of ETFE film and steel arch is both investigated by experiments and analysis. The experiments show the large effect of friction contact on the ETFE films during the extending boundary construction work. The finite element procedure, which combined the nonlinear geometry of membrane elements and nonlinear boundary condition of friction contact, therefore, was proposed. The node-to-node contact elements or Gap elements whose symmetric stiffness matrix could be used to model the friction contact in nonlinear geometry analysis. The comparison between the experiments and analysis illustrated the accuracy and applicability of the proposed method. However, the assumption of local coordinate of Gap elements should be studied more in the future in order to have a better evaluation for the friction contact.

Acknowledgements-The support for the experiments from Taiyo Kogyo Corporation is gratefully acknowledged.

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摩擦接触条件を考慮した ETFE 膜を用いた膜構造の非線形有限要素法

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梗　概

フレームに支持された膜構造タイプでは、膜にプレストレスを導入する技術の一つが境界を延伸することである。この場合、膜と支持されたフレームの摩擦接触が観察された。したがって、摩擦接触を考慮した膜構造の解析手法が不可欠である。

本論文では、膜要素の非線形幾何と摩擦接触条件を含む非線形有限要素法について述べる。この分析では、膜要素とフレーム要素との間の摩擦接触を Gap 要素で置き換えた。提案手法を適用して、並行アーチでの筋アーチと ETFE フィルムとの間のすべりを強制変位条件下で調べる。提案された手順の精度と適用性は、実験によって確認された。

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